MTM: Casimir Effect and Gravity Wave Detection

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***Abstract*-MTM (Metamaterial) is a negative index composite structure, involves multi-disciplinary engineering challenges and opportunity. This paper reports MTM inspired structure for analyzing and Casimir effect and GW (Gravity Wave) detection.**

I. INTRODUCTION

MTM (Metamaterial) is an artificial composite structure, exhibits negative index ($n = -\sqrt{\epsilon\mu}$; $\varepsilon < 0$, $\mu < 0$, where n is refractive index, ε is electric permittivity of medium, and μ is magnetic permeability of medium) characteristics. Figure (1) shows the typical characteristics of the medium that explains the properties of natural and artificially engineered composite material [1]. Figure (2) shows the MTM inspired Grapheme nano-ribbons, which possess topology-induced desirable high Q-factor components and desirable electromagnetic properties. Unlike conventional materials, which interact with EM (electromagnetic) wave based on their chemical composition, the properties of NIM (negative index material), and known as MTM come from their geometric topology structure.

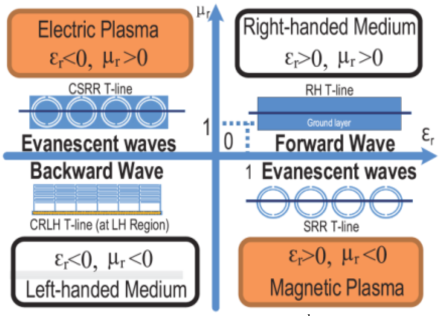

Fig. 1: A typical representation of the medium: 3rd quadrant explains left-handed medium, NIM (negative index) material properties [1]

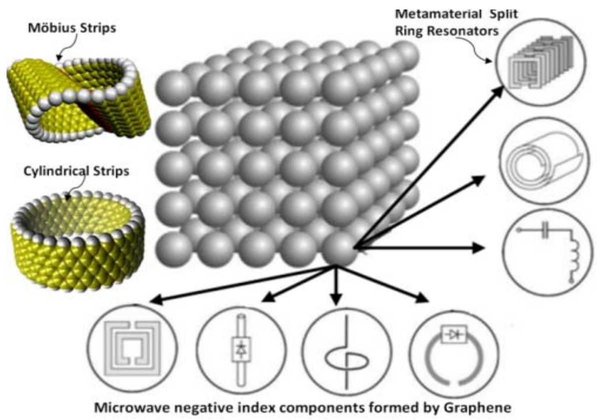

Fig.2: Möbius-Strips structure formed by Grapheme nanoribbons exhibits negative index properties (ε<0, μ<0), well suited suitable for high quality factor microwave frequency components

MTM a non-homogeneous material structure, characterized by effective constitutive parameters: $\varepsilon_r = n/z$, $\mu_r = n \times z$, which gives a direct logical explanation of medium. However, determination of wave impedance 'z' (normalized impedance) of MTM structure is challenging because *it's* value depends on surface termination and overall size of the structure [1].

We can possibly determine index (n) and wave impedance (z) of non-homogeneous periodic MTM structure but requires , prior information about termination of unit cell, phases and amplitudes of the transmitted/reflected wave from the MTM structure. The problem is if the structures are not symmetric along the wave propagation direction, lead to two different values of 'z' corresponding to two incident directions of propagation. This creates uncertainty in the computation of 'z', ε and μ, increases as the ratio of MTM unit cell size to wavelength increases [2]-[3].

II. RETRIEVAL OF MTM'S EFFECTIVE PARAMETERS (ϵ, μ)

SRR (Split Ring Resonator) is considered as a simplest unit MTM cell. Figure (3) shows the typical SRR unit cell structure for a theoretical formulation that gives brief insights about physical properties and geometrical parameters. The analytical values of constitutive parameters ($\varepsilon_{zz}, \mu_{yy}, \xi_0$) is given by

$$\varepsilon_z = 1 + \frac{1}{\varepsilon_0 a^3}\left\{\varepsilon_0 \frac{16}{3} r_{ext}^3 + 4 d_{eff}^2 r_0^2 C_{pul}^2 L\left(\frac{\omega^2}{\omega_0^2 - \omega^2 - i\omega\gamma}\right)\right\} \quad (1)$$

$$\mu_y = 1 + \frac{\mu_0}{a^3}\left\{\frac{\pi^2 r_0}{L}\left(\frac{\omega^2}{\omega_0^2 - \omega^2 - i\omega\gamma}\right)\right\} \quad (2)$$

$$\xi_0 = -i\sqrt{\frac{\mu_0}{\varepsilon_0 a^3}}\left\{-2 i r_0^3 d_{eff} C_{pull}\frac{\omega_0^2}{\omega}\left(\frac{\omega^2}{\omega_0^2 - \omega^2 - i\omega\gamma}\right)\right\} \quad (3)$$

where $d_{eff} = c + s$, $\gamma = \frac{R}{L}$, $r_0 = r + c + s/2$ and $r_{ext} = r + 2c + s$; L and C_{pull} are SRR total inductance and capacitance between the rings. As depicted in Figure (3), SRR acts as a resonant LC circuit, frequency of the resonance is given by

$$\omega_0^2 = \frac{1}{\left(\frac{1}{2}\pi r_0 C_{pull} + C_{gap}\right)L} \quad (4)$$

L and C_{pull} is given by analytical equations [4]. C_{gap} is the capacitance of the gap illustrated in Figure (3), for narrow gap, its capacitance can be theoretically estimates reported in [5]

$$C_{gap} = \frac{\varepsilon_0 ch}{g} + C_0 \text{ with } C_0 = \varepsilon_0(h + c + g) \quad (5)$$

For example, the dimension of SRR is: a = 3 mm, r = 0.74 mm, c = 0.2 mm, s = 0.1 mm, g = 0.2 mm, t = 0.3 mm, h = 35 μm, ε_0=8.8541×10^{-12}F/m. From (1)-(5), L = 3:9287× 10^{-9} H, C_{pul} = 1:6806 × 10^{-11} F, C_{gap} = 4:155 × 10^{-15} F and R = 0:4901 Ω at the frequency of resonance.

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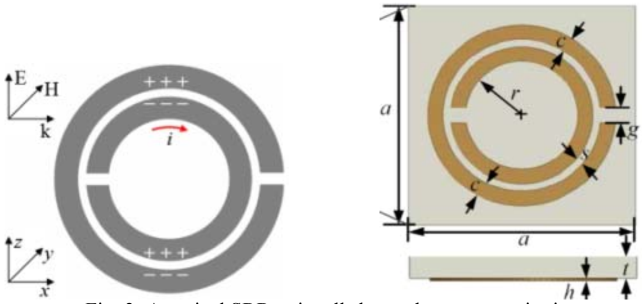

Fig. 3: A typical SRR unit cell shows the asymmetric rings

The numerical values obtained using (1)-(5) are approximate, resulting poor accuracy of analytical method, errors increases for thicker structures. The alternative technique is based on S-parameters extraction.

There are three imperative methods generally used for the determination of effective constitutive parameters based on applications and acceptable limit of errors [3].

The first method is to numerically calculate the ratios of the EM (electromagnetic) field inside MTM's structure but this approach is good for numerical simulations, and not for experimental measurement [6]. The second method calculates the effective constitutive parameters by using analytical models of MTM's structures including numerical averaging of fields. But this technique also not suitable for complex structures such as superconducting MTM. The third method is a retrieval technique based on the inversion of scattering data (S-parameters) of a finite slab, called the NRW (Nicolson–Ross–Weir) procedure [7]-[9]. The problem with NRW method arises in case when topology driven MTM's structure undergo asymmetric reflections. Smith *et al.* [2] reported a modified approach to resolve asymmetric reflections concern with the help of averaged value of reflection coefficients. As we know that isotropic medium model cannot replace this property, hence it is important to understand the EM dynamics of SR inspired MTM medium that intrinsically possess reflection asymmetric structures ($S_{11} \neq S_{22}$) [10].

III. MTM: Emerging Applications

The promising opportunity is in the area of detection of weak signal and signal processing where the space-time cloak serves as a means of prioritizing data channels, instead of theoretically effort to combine space-time and spatial cloaks. The topology inspired Möbius transformation [11] MTM structure is attention-grabbing for a relativistic case, including Casimir force of attraction/repulsion in vacuum.

A. Casimir Effect (Force from nothing)

The Casimir force as illustrated in Figure (4) originates from the interaction of the surfaces with the surrounding electromagnetic spectrum, exhibits dependence on dielectric properties of surfaces and the medium between. Casimir forces between macroscopic surfaces have the same physical origin as atom-surface interactions and those between two atoms or molecules (van de Waals forces), because they originated from quantum fluctuations.

In general, Casimir forces between macroscopic surfaces entail separations typically > 0.1um where retardation plays an essential role, while van der Waals forces refer to separations < 0.01um where retardation is insignificant. Figure (4) shows the practical evidence of Casimir force 'F' on parallel plates kept in vacuum, the effective force $F \propto A/d^4$, where A is the area of plate and d is the distance between the plates [12]. Advances in theoretical studies and experimental techniques have enabled examination of the Casimir force beyond the original configuration of two parallel perfect metal plates. Novel materials and shapes of the interacting bodies enable new opportunities for applications and, at the same time, pose new open questions. On the theoretical side, MTM-Inspired structure can produce a powerful Casimir Effect, which will allow us to transport the matter; this implies that we can use this effect to attract or push away physical matter in principle. The significantly greater complexity of the Casimir force potentially allows greater opportunity for neutralization or for use of Casimir forces to partially cancel Van Der Waals forces. Figure (5) depicts the polaritonic characteristics of the Casimir effect between conventional Metal and MTM structure. It can be seen that polaritonic involvement causes repulsive Casimir force between Metal and MTM structure. For example, L ≥ $\lambda_r/5$ binding TM polariton govern at shorter distance, inundated by joint repulsion due to anti-binding TM and TE polaritons. This explains that for a hybrid arrangement, surface plasmons are decisive in determining strength and sign of the Casimir force of interactions. Figure (6) shows the typical example of levitating mirror (The repulsive Casimir force of a Metamaterial may balance the weight of one of the mirrors, letting it levitate on zero-point fluctuation).

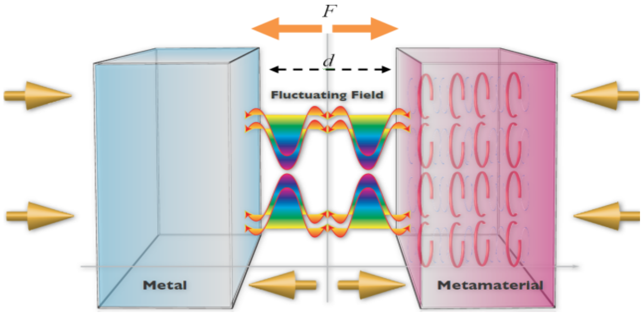

Fig.4. Repulsive Casimir force 'F' on parallel plate kept in vacuum, effective force F ∝- A/d^4, A is the area of plate, d is the distance between the plates.

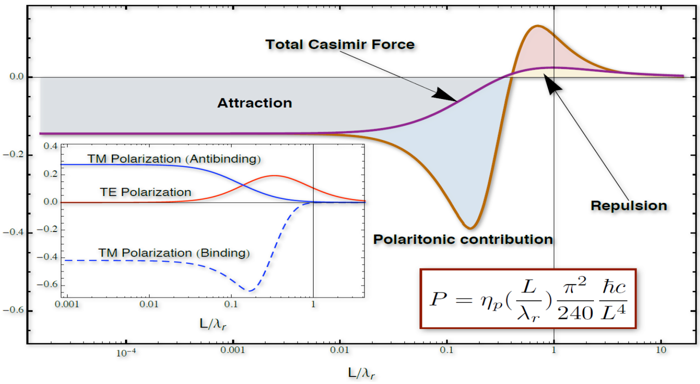

Fig. 5: Typical polaritonic plots between Metal and MTM -Inspired mirror.

The explanation of anti-gravity or levitation effect illustrated in Figure (6) can be given based on analogy vibration modes of violin strings in acoustics. In acoustics, vibration of a violin strings broken down into a grouping of normal modes of oscillation, defined by the distance between the ends of the string. Similarly, oscillating EM fields can also be described in terms of these modes such as different possible standing wave fields in a vacuum inside metal box. According to classical theory, if there is no field in the box, no energy should be present in any normal mode. On the contrary, quantum theory predicts that even when there is no field in the box, the vacuum still contains normal modes of vibration that each possesses a tiny energy, called the zero-point energy. Based on these postulates, number of modes in a closed box with its walls very close together would be restricted by the space between the walls that would make the number smaller than the number in the space outside. Hence, there would be lower total zero-point energy in the box than outside. This difference would produce a tiny but finite inward (metal-metal) force of attraction and outward repulsive force on the walls of the box when one of the surfaces possesses negative permittivity. This approach can resolve the stickiness problem, leading to new material and fabrication methods in next generation electromechanics. MEMS based electronic devices offer inexpensive high performance solutions. But reliability issues arising from stiction in MEMS switching devices, prevents the technology for high frequency applications.

B. *Casimir Effect: Dark Energy*

There are 4- elementary force of nature (gravitational, electromagnetic, strong nuclear, and weak nuclear), which can be described by dynamics of their field, only gravity and electricity have long range, follows inverse square laws. The Casimir effect in presence gravitational fields can demystify many mystery of nature, it is noticed that the effect of the curved space-time ambiance in universe would have on Casimir energy. Universe has matter, radiation, the Hubble horizon and other forms of artificial boundaries in it, and boundaries (like metal plates in electromagnetism) can cut off some of the allowed modes of quantum fluctuations, and lead to a real force: the Casimir effect. Could this same effect, which exists for all the forces, not just electromagnetism be responsible for dark energy? The assumption of expansion of the universe accelerates, but scientific communities believe driving force behind this acceleration is “dark energy”. Theoretical analysis confirms that about 70 percent of the total mass in the universe must be contributed to this mysterious energy (dark energy), which cannot be measured easily.

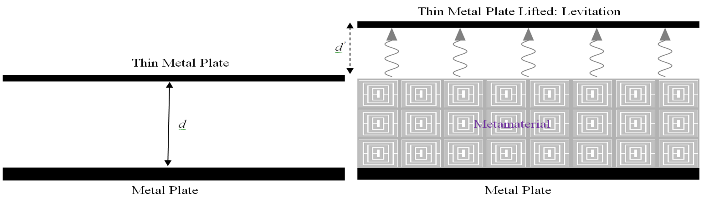

Fig. 6: Levitating mirror: repulsive Casimir force of MTM balance weight of one of the mirror allowing levitate on zero-point fluctuation.

One of the fascinating information about “dark energy” is that it seems to act with enormous force over the vast empty spaces in our universe, pushing space apart, while not having any measurable effect on the scale of planets or even the solar system. The adaptive nature suggests that ‘dark energy’ is a particle which interacts stalwartly in a vacuum, while being screened in 'denser' medium such as on Earth or Milky Way.

C. *Casimir Effect: Detection of GW (Gravitational Wave)*

One of the exciting properties of MTM structure is that they can bend light in a way that is mathematically equivalent to the way space-time bends light, allows topological exploration for the realization of low cost gravitational wave detector. Thin MTM superconducting films are predicted to be highly reflective mirrors for gravitational waves at microwave frequencies [13]. Figure (7) shows the gravitational Casimir effect, with a two plate setup, the change in the index of the MTM plates causes the GW to refract, where k represents the wave-vector of incident, transmitted, and reflected GWs, and γ is the corresponding angle w.r.t surface normal [13].

The mathematical representation based on EM-Maxwell Equations of the Einstein’s general theory of relativity can be described by the coupling dynamics of weak Gravitational fields to a slowly moving matter. Assuming asymptotically flat space-time coordinate system of a distant inertial observer, the EM-Maxwell equivalent 4-Equations are:

$$\nabla \times \bar{E}_G = -\frac{\partial \bar{E}_G}{\partial t} \tag{6}$$

$$\nabla \times \bar{B}_G = \mu_G \left(-j_G + \varepsilon_G \frac{\partial \bar{E}_G}{\partial t}\right) \tag{7}$$

$$\nabla . \bar{E}_G = -\frac{\rho_G}{\varepsilon_G} \tag{8}$$

$$\nabla . \bar{B}_G = 0 \tag{9}$$

The gravitational equivalent of the electric permittivity ε_G and magnetic permeability μ_G of free space is given by

$$\mu_G = \frac{4\pi G}{c^2} = 9.3 \times 10^{-27} \tag{10}$$

$$\varepsilon_G = \frac{1}{4\pi G} = 1.2 \times 10^{9} \tag{11}$$

From (6)-(11), value of ε_G is predetermined, suggests Newton’s law of gravitation be recovered from the Gauss equivalent law (9), while the value of μ_G is set by the linearization procedure from Einstein’s field equations. These two constants express the strengths of the coupling between sources (i.e., of masses and mass currents, respectively) and gravitational fields, and are analogous to the two constants ε_0 (the permittivity of free space) and μ_G (the permeability of free space), which express the strengths of coupling between sources (charges and charge currents, respectively) and electromagnetic fields in Maxwell’s theory.

From (6)-(11), the field $\bar{E}_G$ is the gravito-electric field, which is to be identified with the local acceleration g of a test particle produced by the mass density ρ_G, in the Newtonian limit of general relativity. The field $\bar{B}$ is the gravito-magnetic field produced by the mass current density j_G and by the gravitational analogy of the Maxwell displacement current density $\varepsilon_G \left(\frac{\partial \bar{E}_G}{\partial t}\right)$ of the Ampere-equivalent law (7).

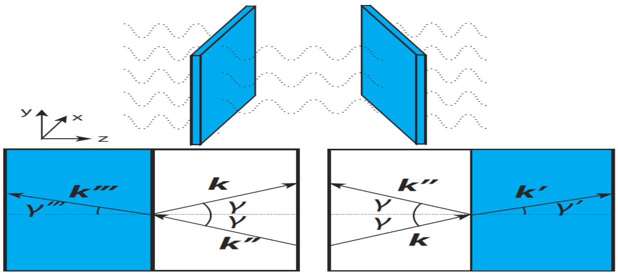

Fig. 7: GW Casimir effect: 2-plate setup, the change in the index of plates causes the GW to refract.

The consequential magnetic-like field $\bar{B}_G$ can be regarded as a generalization of Lense-Thirring field of general relativity. Since these equations are linear, fields will comply with the superposition theory not only outside the source (in vacuum), but also within the matter inside the source, provided the field strengths are amply weak and the matter is adequately slowly moving. Note that the fields $\bar{E}_G$ and $\bar{B}_G$ in the above Maxwell-equivalent equations shall be treated as classical fields, similar to the fields $\bar{E}_G$ and $\bar{B}_G$ in the classical Maxwell's equations.

From (6)-(11), the characteristic GW impedance of free space Z_G, can be given by [14]

$$Z_G = \sqrt{\frac{\mu_G}{\varepsilon_G}} = \frac{4\pi G}{c} = 2.8 \times 10^{-18} \tag{12}$$

Equation (12) represents the characteristic of the vacuum, i.e., property of space-time, independent of matter properties. For example, $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377$ ohms in the EM field, $Z_G = \sqrt{\frac{\mu_G}{\varepsilon_G}} = 2.8 \times 10^{-18}$ dictate a imperative position in all GW radiation coupling dynamics interaction problems.

In reality, impedance of a material object should be lesser in dimension than this awfully minute quantity before any noteworthy fraction of the incident GW energy reflected. The alternative method of GW reflection from macroscopically coherent quantum systems such as MTM-Inspired superconductors can improve the dynamic range of detection.

Peter [15] reported the GW refractive index n_G, which was larger than that generated by just considering the induced quadruple moments, suggesting that his model encapsulates the dominant GW interaction with matter, given as

$$n_G = 1 + \frac{2\pi G\rho}{\omega^2} \tag{13}$$

where ρ is the density of the medium. Minter [16] reported the reflection coefficient of a superconducting film from an incident GW (gravitational wave GW) as

$$r_G = \frac{1}{1+\frac{2\delta^2}{c\times d}\xi} \tag{14}$$

where δ is the EM skin depth of the superconducting film and d the film thickness. From (14), the Gravitonic contribution to Casimir pressure for superconducting lead (Pb) of thickness d = 2nm at zero temperature is plotted in Figure (8). The EM skin depth of Pb is δ = 37 nm. This result is compared with the photonic contribution to the Casimir pressure of superconducting lead.

The EM reflection coefficient is given by [16]

$$r_E = \frac{1}{1+\frac{2\lambda\delta^2}{c\times d}\xi} \tag{15}$$

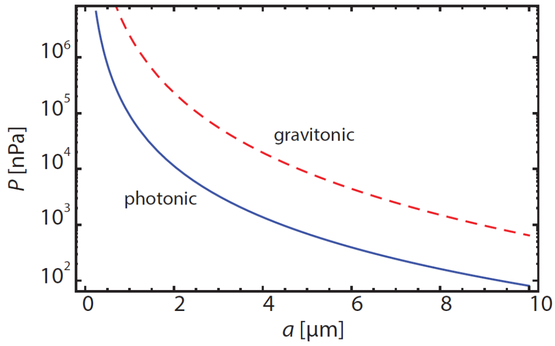

Fig. 8: CAD simulated plots of Gravitonic (dotted line) and Photonic (solid line) contributions to the Casimir pressure of parallel plates kept in vacuum

where λ = 83 nm is the coherence length. The photonic contribution to the Casimir pressure is calculated by (15).

IV. Conclusion

Magneto electric-couplings can be a source of new behavior in Casimir systems, Metamaterial Casimir repulsive effect can lead to anti-gravity and low cost solution for levitation. The Metasurface made of MIM can provide nearly infinite group delay, very helpful in understanding Einstein Precession, Geodetic Effect and new evidence that requires refining our understanding of the relativistic corrections to Newtonian Celestial Mechanics.

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